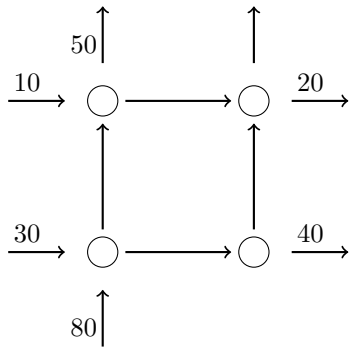
MTH 309 - Activity 2
Systems from Applications

1. In the City of Buffalo, we have many one way streets and crazy intersections. Consider the following diagram of a block of a neighborhood near Delaware Park.




   Arrows indicate traffic direction, and numbers indicate the number of cars per hour on that stretch of road. The circles indicate intersections.

   Due to availability of equipment, we can only measure 6 traffic flows at a time, so we need to determine the remaining traffic flows.

   (a) Write a system of equations that models the traffic flow.
   (b) How many solutions does the system have?
   (c) Find the missing traffic flows.
   (d) Suppose a water main break closes the southeast intersection. Write a solve a system of equations for the traffic flows after the closure.

2. Linear algebra is used in many subjections including chemistry where systems of equations can help balance chemical reactions. Consider the burning of gasoline (a mixture of octane and ethanol):

$$C_8H_18 + C_2H_5OH + O_2 \longrightarrow CO_2 + CO + H_2O.$$

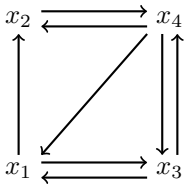
   This reaction has no coefficients and is unbalanced–there are way more carbons on the reactants (left) side.

   (a) Add coefficient variables to the reaction, then write a system of equations that models a balanced reaction.
   (b) Without doing any computations, how many solutions should this system have?
   (c) Determine the number of solutions and solve the system.
   (d) Now balance the reaction $P_4 + NaOH + H_2O \to NaH_2PO_2 + PH_3$.

3. In economics, systems can be used to determine optimal production levels. Consider a factory that makes three products A, B, and C. Product A uses 5 resources, costs \$5 in parts and labor, and retails for \$7. Product B is more labor intensive, it uses 3 resources, costs \$10 to make, and retails for \$15. Their biggest product, product C uses 14 resources, costs \$22 to make, and retails for \$42. The company has a total of 1300 resources, has a budget of \$2550, and would like to hit a revenue goal of \$4000.

   (a) Write a system that models the companies goals.
   (b) How many of each product should the company make?

4. The PageRank algorithm. Consider a network of webpages that link to one another that results from a Google search. We know that Google ranks these webpages in order of relevance, but how do they determine the order? We would like to assign a value to each webpage (its rank), so we assign a variable to each webpage and solve a system of linear equations to determine their values.



Arrows indicate links from one page to another.

Not all links are created equal. Before we can get a system of equations, we need to assign values to each of the link. To do this,

i. count the number of outgoing link from a webpage, say there are $n$ of them

ii. assign each of those outgoing links a value of $1/n$ times the rank of the webpage

iii. do this for all the webpages.

Now we can get a system of equations. For each webpage, the page rank is the sum of the values of incoming links.

(a) Write a system of equations in standard form for the page ranks. Is there anything special about this system?

(b) Find a solution for the page ranks of the network above.

(c) Now add an extra equation that stipulates that the sum of the page ranks is equation to 1 and solve again.

5. Consider three points in the plane (1,1), (2,6), (3,5). There is a unique parabola passing through all three points represented by the quadratic equation $x_2 = ax_1^2 + bx_1 + c$ for a particular triple of values $(a, b, c)$. We would like to determine those values.

(a) Determine the values of the coefficients $a, b, c$.

(b) Find the coefficients of the unique cubic curve that passes through the points (-2,8), (-1,0), (1,2), (5,1).