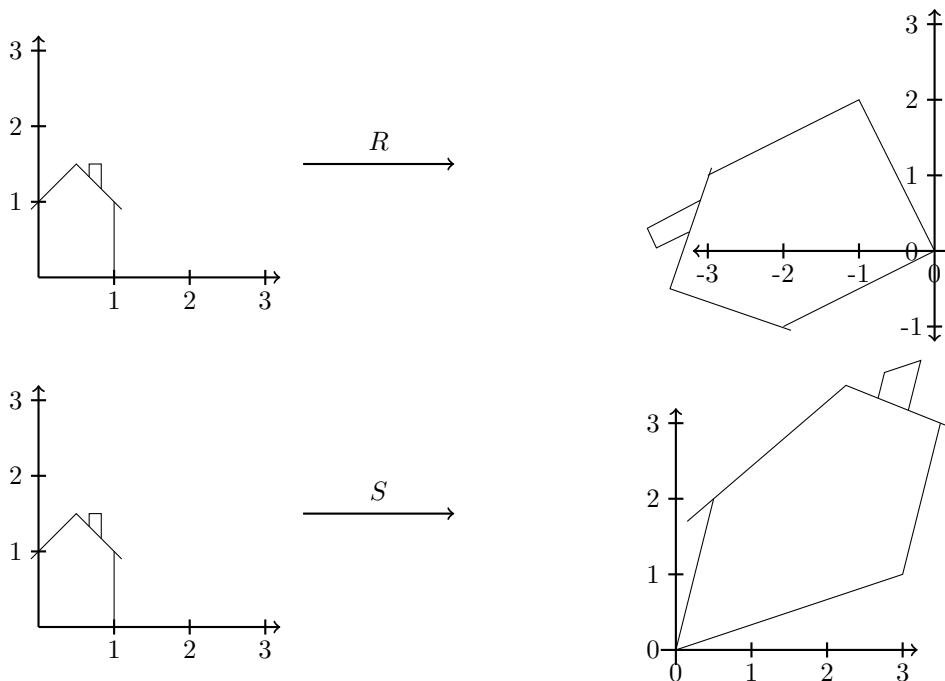# MTH 309 - Activity 8
Matrix Algebra

1. Find the matrix representations for the two transformations pictured below.

2. Find the matrix representation for the transformation $(R + S)(v) = R(v) + S(v)$. What is the relationship between the three matrix representations for R, S, and $R + S$?

It is often convenient to think of evaluation of a linear transformation as multiplication of the matrix representation by the input vector. That is if the linear transformation T has matrix representation A, then we define $Av = T(v)$.

Example: Consider the linear transformation $T(x, y) = (3x - 4y, x + 5y)$, whose matrix representation is $\begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}$. By the above definition,

$$\begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix} = T(4, -7) = \begin{bmatrix} 40 \\ -31 \end{bmatrix}$$

In general,

$$\begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = T(x, y) = \begin{bmatrix} 3x - 4y \\ x + 5y \end{bmatrix} = x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} -4 \\ 5 \end{bmatrix}.$$

This shows us that we can think about Av as a linear combination of the columns of A with the coefficients coming from v:

$$Av = v_1 A_{*1} + v_2 A_{*2} + \cdots + v_n A_{*n}.$$

3. Find the matrix representation for the transformation $(R \circ S)(v) = R(S(v))$. What is the relationship between the three matrix representations for R, S, and $R \circ S$?

4. Do you expect the matrix representation for $S \circ R$ to be the same as the one for $R \circ S$? Explain your reasoning.

5. Find the matrix representation for the transformation $(S \circ R)(v) = S(R(v))$. Do you get the same matrix as for $R \circ S$?

6. What does this tell you about the matrix algebra involved?