MTH 309 - Activity 10
Systems of Differential Equations

Consider a simple 1-dimensional differential equation $\frac{dx}{dt} = ax(t)$, the solutions to this equation look like $x(t) = ce^{at}$ where c is a constant.

Now consider the system of differential equations

$$\frac{dx}{dt} = ax(t) + by(t)$$
$$\frac{dy}{dt} = cx(t) + dy(t).$$

1. Write this system as a matrix-vector equation. Do you notice any similarities to the simple 1-dim'l system?
2. Make a guess as to a solution. What would it look like if we adapted the 1-dim'l solution to work in this 2-dim'l system?
3. Plug your guess into the matrix-vector equation. What do we get as a result?

Solutions to differential equations form subspaces of the vector space of differentiable functions. A 1-dim'l equation $\frac{dx}{dt} = ax(t)$ will generate a 1-dim'l subspace of solutions, the span of e^{at}.

Suppose the 2-dim'l system from before has two linearly independent solutions $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$.

4. Show that the general linear combination $\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)$ is also a solution.
5. Find the general solution formula for the following system of equations.

$$\frac{dx}{dt} = 3x(t) - y(t)$$
$$\frac{dy}{dt} = 5x(t) - 2y(t)$$