MTH 309 - Activity 10
Eigenvalues and Eigenvectors

1. Go to https://www.geogebra.org/m/mdvN0HTt. There you will find an interactable graph containing two vectors, a unit vector (length 1) input that you can drag around, and the corresponding output vector for the linear transformation represented by the matrix. Drag the input vector around the circle to see what happens to the output vector.

   (a) As you drag around the input vector, do any of the inputs stand out in terms of what is happening to the output?

   (b) For the inputs that stand out, what is the relationship between the input and the output?

   (c) Translate your answer to part b into an equation(s).

2. Change the matrix to each of the following and write down your observations about these special input vectors.

   i. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

   ii. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   iii. $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

3. Now think about the following linear transformations. Without doing any calculations, are there any input vectors that have a special relationship with the corresponding output vectors?

   i. Reflection across the line spanned by vector $\mathbf{v}$.

   ii. Projection onto the line spanned by vector $\mathbf{v}$.

   iii. Rotation about the origin by θ radians counterclockwise.