

Proficiency Exam 3 - Linear Transformations

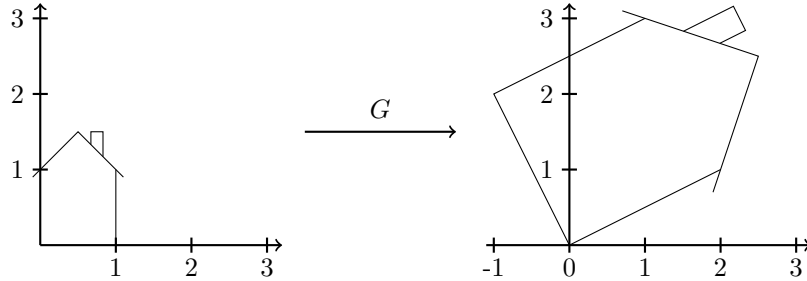
You will have 30 minutes to complete the exam. You may use a calculator, but you must show all steps done to get full credit for completing the problem. This means that if you use your calculator for anything other than arithmetic, you must indicate on your test paper what you did on the calculator.

1. Is the function  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  given by

$$F(x_1, x_2) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 2x_1 - 3x_2 \end{bmatrix}$$

a linear transformation? If it is, prove it. If it isn't, provide a counterexample that shows it is not.

2. Find the matrix representation of the linear transformation pictured below.



3. (TRUE or FALSE) Consider the statement and decide if it is true or false. If true, provide reasoning. If false, provide a counterexample.

“If  $T: \mathbb{R}^m \rightarrow \mathbb{R}^n$  is onto, then  $m \leq n$ .”

4. Is the following linear transformation onto?

$$T(x_1, x_2, x_3) = \begin{bmatrix} x_1 - 3x_3 \\ 2x_2 + 4x_3 \\ -x_1 + 3x_2 + 7x_3 \end{bmatrix}$$