Proficiency Exam 6 - Vector Geometry

You will have 30 minutes to complete the exam. You may use a calculator, but you must show all steps done to get full credit for completing the problem. This means that if you use your calculator for anything other than arithmetic, you must indicate on your test paper what you did on the calculator.

1. Compute the vector $\mathbf{n}$ which is orthogonal to $\mathbf{w} = (0, -0.6, 0.8)$ and such that $\mathbf{v} - \mathbf{n} \in \text{span}(\mathbf{w})$ where $\mathbf{v} = (1, 2, -1)$.
   Hint: draw a picture.

2. For what value of t will the set of vectors below be orthogonal?

$$\left\{ \begin{bmatrix} t \\ 0 \\ 0 \\ 2t \end{bmatrix}, \begin{bmatrix} 2 \\ t+1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2t \\ t \\ t-6 \\ t \end{bmatrix} \right\}$$

3. (TRUE or FALSE) Consider the statement and decide if it is true or false. If true, provide reasoning. If false, provide a counterexample.

   "Let W be a subspace of $\mathbb{R}^3$ with basis $\{b_1, b_2\}$. The projection of a vector v onto the subspace W is given by the formula $\text{proj}_W(v) = \text{proj}_{b_1}(v) + \text{proj}_{b_2}(v)$."

4. Find an orthonormal basis for

$$W = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -6 \\ 1 \end{bmatrix} \right\}.$$